# MATHEMATICAL OPTIMIZATION OF DATA CHANNELS

## RÁDIÓCSATORNÁK ADATÁTVITELÉNEK MATEMATIKAI OPTIMALIZÁLÁSA

GULYÁS Attila

(ORCID: 0000-0001-5774-5757)

attila.gulyas@mil.hu

**Absztrakt**

*A kognitív rádiócsatorna spektrális modellezése lehetővé tette a szűkösen rendelkezésre álló elektromágneses spektrum hatékonyabb felhasználását megoldási lehetőségek koherens kidolgozásával. A dinamikus rádiófrekvenciás spektrummenedzsment kiemelten szükséges a keskeny- és szélessávú jelátvitel szempontjából, a bithibák elkerülése és a csatornák, részcsatornák teljesítményallokációjának optimalizálása érdekében. E tudományos közleményben már létező műszaki eljárások matematikai modelljét vizsgálom a jelátvitel optimalizálása érdekében.*

***Kulcsszavak:*** *Gauss-csatorna, szélessávú adatátvitel, Iterative Water-filling eljárások (IWFA)*

**Abstract**

*The cognitive radio channel allocation is envisioned to solve the challenge of electromagnetic spectrum scarcity focusing on radio frequency's data networking in the ever-changing radio environment. Dynamic electromagnetic spectrum management is crucial to avoid decreasing the bit rate and allocate the appropriate channel power in cognitive radio networks. This scientific essay is about the mathematical modelling of existing technical solutions for seamless communications networking.*

***Keywords:*** Gaussian multiple-access, multi-channel, Iterative Water-filling Algorithm (IWFA)

# INTRODUCTION

The electromagnetic spectrum became one of the finest sources available for the 21st century. Provisions for a large amount of information and preparation for data collection have focused for the innovative solutions of data on the field of digital broadcasting. However, the standardized utilization of the frequency bands regulated nationally [1] and/or by the European Union and the North Atlantic Treaty Organization is aperiodic and this, in many cases, leads to an imbalance in the operational use of available channels.

In military terms, tactical and operational radio broadcasting systems should primarily be designed for the combined use of short-wave and ultra-short wave frequency bands [2][3] with a particular attention to the joint multi-channel operations (parallel channel and sub-channel usages) as essential practice in the period of multinational implementation.

One of the most important requirements for the civilian and military radio networks is to maximize data rates by ensuring signal stability (minimizing the errors of bits) and keeping the channel capacities assigned to the transmission channels at optimal value [4][5]. The latter is of particular importance in view of the limited availability of energy sources and modulation processes. A further condition is to enhance the communication capability of the communications channel to create the ability to adapt to the ever changing and increasingly polluted, noisy electromagnetic environment (to be adaptive) [6].

I consider the modeling of the data transmission network is vital for satisfying this conditionality so I review a mathematical solution in connection with the modulation of the signal designed for transmission and the associated channel power allocations through a simplified scheme. In addition, I examine the practical application of orthogonal frequency division multiplexing (OFDM) with the support of the water filling algorithm as a performance optimizing mathematical model to optimize orthogonal sub-channel performance.

## TECHNICAL CHARACTERIZATION OF RADIO FREQUENCY DATA TRANSMISSION CHANNELS

In the modern digital battlefield it becomes increasingly difficult to interpret the obvious differences between the analogue and digital communications subsystems in the command and control networks. For complex broadcasting services the following requirements can be defined [7][8] as follows: full availability of surveillance and reconnaissance systems, coordinated access to military command and control information systems, point-to-point and point-to-multipoint voice and data link planning with analogue and digital redundant radio channels (satellite, short, ultra-short, microwave). Moreover, the broadcast channels can be able to provide e-mailing and video teleconferencing (VTC), neutral and friendly force tracking (FFT), receiving and continuous processing of remote sensors' control signals; processing analogue and digital channels for aircraft engaged in unmanned surveillance and reconnaissance for tactical and operational tasks. It should also be possible to establish a point-to-point and point-to-multipoint voice and data connection on analogue and digital redundant cable and wireless channels.

Running these services is a major challenge for both the network planners and the network operators. The primary objective is to ensure the same quality of networking services used in peacetime trainings, exercises and operations in the operational area that requires forward-looking dynamic network planning and optimized process management [9][10]. The requirements for the tactical subsystems of network services can thus be ensured by a coherent network design with high redundancy in the available frequency range of the electromagnetic spectrum optimally designed by mathematical calculations.

## CHARACTERIZATION OF MULTIPLE ACCESS DATA CHANNEL

Military radio broadcast systems can be characterized as a network of multiple access with simultaneous distribution by multiple transmitters and multiple receivers, where the access of recording and the processing of data is based on the principle of multiple inputs – multiple outputs rules (MIMO) [11][12][13]. During the radio broadcasting the basic requirement is that the individual terminal reproduces the electromagnetic waves emitted on the transmitter by neutralizing the disturbances of transmission channels.

Many studies have been completed in the last years focused on radio broadcasting. Based on the theoretical approach, a study of *Gaussian channel* [14] is justified in order to plan the whole channel capacity allocation. If the Gauss channel input ($i^{th}$) is $X_i$ then the output signal is $Y_i=X_i+Z_i$ where $Z_i$ is the independent equally distributed value of the variables in the channel [15][16]. In order to determine the transmission capacity ($C$) of the broadcast channel, it is advisable to first quote the analog of the discrete definition by considering that the absolute continuous input distribution needs to be firstly investigated then the information content of the input and output can be interpreted by minimizing the additive components [17][18].

$$C = I(X;Y) \tag{1}$$

in

$$I(X;Y) = h(Y) - h(X|Y) = h(Y) - h(Z)$$

$h$ = channel entropy

As consequence from this identity that channel capacity, theoretically based only on a mathematical approach, can add infinite value [19][20]. However, channel capacity is clearly limited in real conditions. It is therefore advisable to place the channel entropy under some kind of restriction to limit and/or to define the upper limit. Maximizing the value of $h$ by centering the value of the input signal distribution variant ($E = 0$), it can be accomplished using the following mathematical form (Formula 2) [21][22].

$$E(Y^2) = E(X^2) + E(Z^2) \leq P + \sigma^2 \tag{2}$$

where $P$ is the channel power, $\sigma$ is the standard deviation of entropy (Gaussian distribution). In conclusion, the transmission capacity of the Gaussian channel to be tested can be calculated as follows (Formula 3) [23]:

$$C(P) = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right) \tag{3}$$

The determination of the mathematical value of the channel performance enables the optimal transmission of the radio transmission system in order to be able to safely produce the transmitted signal on the output side to correct possible bit errors and to maintain the authenticity and the level of qualification of the network. Taking into account the values in Formula 3 separately, the calculated capacity becomes a new meaning in the technical implementation of some modulation modes when it is possible to streamline and transmit high speed data streams to smaller data packets by optimizing the available and limited channel capacity. In the following I present a modulation process that provides the basis for high-speed data transmission by a large number of users (theoretical data rate for downloading is 100

Mbit/s and 50 Mbit/s uploading) capable of network data communications for fourth generation mobile phones (smart devices, LTE 4G)[1]) [24][25][26][27].

## MODULATION METHOD FOR SPECTRUM MANAGEMENT

Among the complex tasks of the radio equipment, it should be emphasized that the software of the radio equipment must continuously monitor the designated frequency range (band), memorize and record the available broadcast-free channels and sub-channels, analyze the radio frequency interference levels. In other words they need to continuously monitor radio channel capacities, available bandwidths and last but not least accomplishes the spectrum management by allowing optimal allocation and the usage of radio channels among a large number of users.

The basic idea of the opportunistic radio network were introduced in the article of Joseph Mitola in 1999 for developing cognitive radios and optimized radio channels [28]. Mitola explained that the cognitive radio of the future is capable of operating at system-wise and the radio channels must be a continuous monitoring element capable of using radio channels without disturbing the normal operation of primary (incumbent) systems. These operational needs can already be met on the basis of modern software-controlled radio equipment [29]. Multiple access to radio channels, modern modulation technologies and the development of integrated circuits have made it possible to accelerate the spread of the new trend nowadays.

The newer generation of wireless telecommunication networks, WLAN[2] systems and digital broadcasting both demand the widest possible use of the limited radio spectrum, which is unthinkable without increasing the efficiency of multiple accesses. When designing cognitive radios, modulation modes are selected based on the following properties: complexity of signal processing; spectral properties of the generated signal (bandwidth, channel-crosstalk); the value of the peak factor; channel synchronization and Inter Symbol Interface (ISI).

The public engineering solutions are mostly based on OFDM access or different versions of that procedure. The technical basis of the OFDM as a multiplexing process was laid down in the 1950s [30], but the effective application and spread of the software was made possible by the appearance of software-controlled radios. OFDM is based on the idea that the transmission of a high data rate data stream on a radio channel can be accomplished by dividing the data stream into multiple lower speed data channels and by individual, orthogonal, digital modulation of their carrier frequencies, independent of other carriers [31]. With this method the symbol values associated with the transmission of the carriers in the radio channel will be multiples of the original, but the transmission of the sub-channels becomes more resistant to the time scatter, resulting from the wave propagation as well as the emerging noise. The digital signals to be transmitted through the radio channel to the receiving site (receiving antenna) are not only direct, indirect or through multiple reflections so interference between the received symbols must be counted.

If the maximum time difference between channels is $t_{max}$ and the time of the transmitted data symbol is $T$, the value of the symbol interference is as follows:

$$\text{ISI} = \frac{t_{max}}{T} \qquad (4)$$

---

[1] LTE 4G– Long-Term Evolution 4th Generation. The OFDM technique implemented in the physical layer provides an efficient solution for the transmission of high speed (broadband) data including multi-path propagation in the radio channel, significantly increasing the data rate available. OFDM transmission can also support operation on a complex (even non-continuous) spectrum arrangement.
[2] WLAN – Wide Local Area Network

For an $N$-channel multi-carrier system, the data rate ($D_r$) of a sub-channel is the ratio of the total channel rate to the number of channels. Focusing on that, the partial channel symbol time ($T_r$) can be calculated with the following relationship [32]:

$$T_r = \frac{1}{D_r} \tag{5}$$

It follows from mathematical substitutions that the sub-channel symbol time is the N-times of the original symbolic, i.e. the value of the ISI sub-channel [33, p. 23.]:

$$ISI_r = \frac{t_{max}}{T_r} = \frac{t_{max}}{T \cdot N} \tag{6}$$

This correlation points out that the reduction of the inter-channel interference for N-ed and the correct selection of the number of sub-channels can significantly reduce the adverse effects of multi-wave wave propagation. Keeping the value of the ISI low is one of the priority tasks of the receiver but the number of carrier frequencies can be as many as thousands, taking into account the limitations of real technology implementation.

Modulation and demodulation of multiple carriers can be accomplished by discrete Fourier transformation, fast Fourier transformation (FFT[3]), or its inverse operation (IFFT[4]). In this case, a function ($f_t$) can also be given by N-frequency samples ($f_m$), where m = 0,1,2 ... N-1. The values of $f_m$ are complex numbers and Fourier transformations give them the N-values. Discrete Fourier transformed mathematical form [33, p. 24.] as follows:

$$F_{(n)} = \sum_{m=0}^{N-1} f(k)e^{j\frac{2\pi mn}{N}} \qquad \text{n=0,1,2...N-1} \tag{7}$$

During the signal processing the *Shannon/Nyquist* sampling rules must be followed. The sampling frequency should be double of the highest applied frequency ($f_{mv}$ - sampling frequency, $T_{mv}$ - sampling period, $f_N$ - sample frequency).

$$f_{mv} = \frac{1}{T_{mv}} = 2f_{max} \qquad\qquad f_N = 2f_{max} \tag{8}$$

A symbol of the band-bound signal must be a N-piece with a $T_{mv}$ frequency over a period of T ($\Delta t$ – sampling time, B – bandwith) [33, p. 24.]:

$$\Delta t = \frac{1}{2}B = T_{mv} \tag{9}$$

The time series include the N-pattern (N = 0,1,2,3 ... N-1) of the frequency range. The zero element is the average of the time function. The first element is the first harmonic and the other frequency components are the first harmonic multiples. Thus, the N-band frequency range in the given bandwidth range can be considered symmetric, the Nth pattern being the Nyquist frequency. In this symmetric case, it is sufficient to observe the frequency values in spectral terms to the N/2 sample. This is the frequency value whose definition is particularly important

---

[3] FFT – Fast Fourier-Transformation
[4] IFFT – Inverse Fast Fourier Transformation

for the subsequent error-free restoration of the data signal (IDFT[5]). During the sampling the lower and upper side bars are displayed according to the base band. If our sampling frequency is less than the sample rate, we are talking about bottom sampling where the bottom band and the lower band of the sampling frequency overlap. In the base band malfunction components appear that cause data transmission errors. The elimination of this can be accomplished by correctly selecting the frequency values applied and by using customer-side error correction procedures.

The generation of orthogonal components of OFDM is performed by digital signal processing by modifying (amplitude, frequency, phase) a component of sinusoidal signals. In practice, the most widespread technical solutions are Amplitude Shift Keying (ASK) and Quadrature Amplitude Modulation (QAM). The baseband time function ($u_{as}$) can be written with the following relationship: ($A_k+jB_k$ – complex digital modulation at the k-data block; $e^{j2\pi fvt}$ – carrier) [33, p. 25.]:

$$u_{as}(t) = (A_k + jB_k) \cdot e^{j2\pi f_v t} \qquad (10)$$

Sub-carrier amplitude and phase modulation can be used to convert information to the transmission channel. The time function is [33, p. 25.]:

$$z_{as} = \sum_{n=0}^{N-1} \left\{ a_n \cos(2\pi f_0 t) + b_n \sin(2\pi f_0 t) \right\} \qquad (11)$$

To produce the $u_{as}$ baseband signal, a N-sub carrier frequency is required. The total modulated time formula is the sub carriers modulated sum of functions of separately at the same time. Now we have to examine how the mathematical model of OFDM signal can be used in real signal generation. The broadband signal is generated after performing the following sub-tasks (Figure 1): quantization / coding / overlapping, serial-parallel conversion, modulation and Fourier transform, parallel-serial conversion, and D / A conversion, followed by the base band OFDM signal to a data channel. The quantize/encoder/overlay unit encodes and generates the protection of the broadband stream against transmission errors. The data stream is subdivided into several smaller data channels, then modulated separately, and converted into sample values for time functions using the Fourier transform.
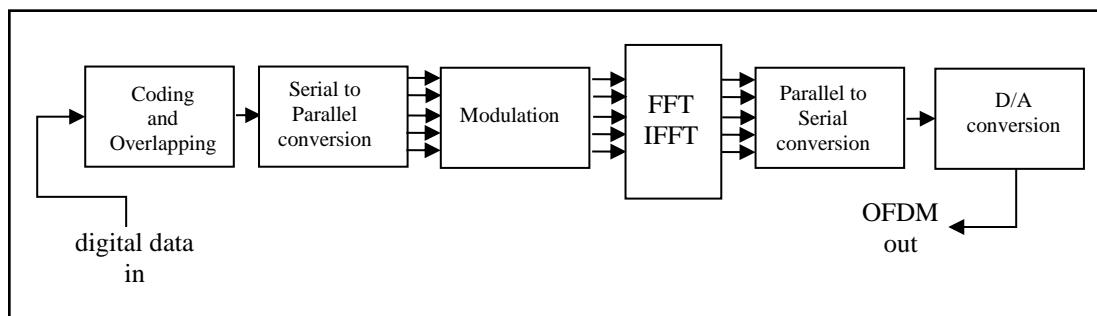


1. ábra. Schema of OFDM process[6]

The purpose of the guard interval is to reduce the effects of fading caused by channel propagation. The discrete sampling values of the time functions of the parallel sub channel thus produced are sequenced, and then the D/A unit generates the base band analog OFDM signal

---

[5] IDFT – Inverse Discrete Fourier Transformation
[6] Edited by the author

which is emitted by the corresponding antenna. By increasing the carrier number the bandwidth utilization improves and in the same time the adjacent channel leakage (spectral power density) decreases. The limit of this technology is the difference between offset and receiver oscillator frequencies (offset), which can cause significant crosstalk between adjacent carrier frequencies. In the design of the modulation system especially for the construction of the amplifier the challenge is to keep the peak factor value and dynamics within defined limits, which is one of the characteristic values of the OFDM sequence. The peak factor (PAPR[7]) value is the ratio of peak to average power of the signal [34]:

$$PAPR = 10\log_{10}\left(\frac{x^2_{n\max}}{\sum_n (x_n)^2}\right) \qquad \text{n=0,1,2...N-1} \qquad (12)$$

The value of the peak factor also depends on the number of carriers. In case of numerous carriers the value of the peak factor may exceed a critical value that limits the physical configuration of the amplifier. Therefore, during the circuit implementation, peak factor reduction techniques are required. Summing up the benefits of OFDM, we can say that the orthogonality of the sub-channels and the use of protective bands significantly reduce the channel interference, making it easier for the sub-channels to equalize the channel. Different versions of the Fourier transformation reduce the number of computing operations and the efficient use of the frequency spectrum to be transmitted can be accomplished by overlapping the sub-spectra.

The disadvantage of the technical process is that the amplitude distribution is of high dynamics therefore the value of the peak factor is high and the stability and coincidence of the carrier frequencies on the transmitter and the receiver side has paramount importance. In addition, attention should be paid to the appropriate level of performance allocation of orthogonal channels to avoid interference with incumbent systems and to maintain opportunistic system constants.

Based on its characteristics and commercial application experience, it can be seen that OFDM can also be well suited for the development of military cognitive radio equipment. From a military point of view, keeping the value of inter-symbol interference defined in mathematical context (Formula 6) is low, as this is one of the basic conditions for ensuring the robustness of the connection. The type of procedures used to modulate sub carriers (Formula 11) determines the stability of the transmission system, for which keeping the peak factor value at the low level of Formula 12 is also a basic requirement.

## MATHEMATICAL SOLUTION FOR SUB CHANNEL PERFORMANCE OPTIMIZATION (VERSION)

In this chapter I will elaborate, which mathematical methods and procedures can be taken into consideration for the proper supply of power to the sub channels due to after modulation of the sub channels, it is necessary to ensure that the N-sub carrier frequency is transmitted with the optimum performance, after examining the sub-channel parameters. In this case the optimal power value's determination means that for the amplifier (transmitting side) the desired sub-channel outputs can be produced physically without interference while processing on the receiving side, the error value (bit errors) of the incremental sub channels remain within the predetermined error limit that can still be interpreted. A technical solution can be the application of Iterative Water-filling Algorithm (IWFA) [35][36], which is widely used in the field of non-military communications systems. Here is how to apply this mathematical model to optimize

---

[7] PAPR – Peak-to-Power Ratio

broadcasting performance. The original idea of the algorithm can be derived from the physical examination of the particles of the spilled water. The ground-level water perfectly fills the available plane due to gravity. This observation and characterization with vectors can also be used in telecommunication technology to investigate the proper feeding of data channels. This is especially important when testing and interpreting systems designed for broadband data transmission when as in OFDM, high-speed data is spread over several orthogonal data channels and each data channel requires optimal power allocation.

The theoretical OFDM channel like the Gaussian channel characterization can be described as follows (Formula 13) [37][38].

$$y_n(m) = h_n x_n(m) + w(m) \qquad n = 0,1,2, N-1 \qquad (13)$$

where y = absolute value of channel output signal, h = channel entropy (gain), x = absolute value of channel input signal, w = channel noise value for each sub-channel. To calculate the power component ($P_n$), determine the data rate (Formula 14) where the noise power density is average ($N_0$) [39].

$$C = \sum_{n=0}^{N-1} \log(1 + \frac{P_n |h_n|^2}{N_0}) \qquad (bit / symbol) \qquad (14)$$

By optimally selecting the value of the power component ($P_n$) in the mathematical correlation counter, the capacity of the data transmission channel and the maximum data rate can be set. Alternatively, the power component value and the channel entropy multiplier determine the data rate. Further analyzing the possible ways to improve performance I make the following clauses as follows.

$$\sum_{k=0}^{N-1} P_n = P_{\text{összes}} \qquad P_n \geq 0, n = 0,1,2...N-1 \qquad (15)$$

Considering that the solution of Formula 14 results in a convex function [40][41] the Lagrange-formula for performance optimization [42][43] can foster the understanding. Introduce the following relationship (Formula 16).

$$\iota(\lambda, P_0, P_1,...P_{N-1}) = \sum_{n=0}^{N-1} \log(1 + \frac{P_n |h_n|^2}{N_0} - \lambda \sum_{n=0}^{N-1} P_n \qquad (16)$$

In this equation "$\lambda$" summarizes the value of the Lagrange formula [44][45][46]. Using the Karush-Kuhn-Tucker Identity [47][48][49] the channel optimization can be achieved by deriving the particular Lagrange-formula by solving the following equations (Formula 17) [37 p. 2070].

$$\frac{\partial \iota}{\partial P_n} = 0 \qquad \text{in case of } P_n > 0$$

$$\frac{\partial \iota}{\partial P_n} \leq 0 \qquad \text{in case of } P_n = 0 \qquad (17)$$

After solving the equations the required power allocation can be described as follows (Formula 18) [37 p.2071][50].

$$P_n = \left( \frac{1}{\lambda} - \frac{N_0}{|h_n|^2} \right)$$ (18)

To sum up, the optimal power allocation – within the given power spectrum – is realized by providing a higher power support (allocation) for a sub-channel that is considered more appropriate (with a higher signal-to-noise ratio) during the software scan. Alternatively, there is a computational relationship between the partial channel data rate (bit rate/symbol transmission) and the sub channel allocation based on Formula 16. The computational load of the control processor must be kept to a minimum according to Formula 18 in order to allow the use of processor capacity to support functions other than its principal functions.

## SUMMARY

As described in the scientific essay the existing mathematical procedures referred can be used to optimize channel performance to implement broadband support modes. The mathematical algorithms are used in the programming of software-controlled radio equipment to ensure that the power supply of the ultimate available power bases and battery packs optimally supports the energy consumption of the sub channels.

The need for point-to-multipoint and multipoint-to-multipoint broadband transmission basically requires the designed and coherent application of network operation algorithms (modulation modes and performance optimization procedures), that support reliably and available electromagnetic interactions in multinational operational environments during international operations, optimizing the use of spectrum and energy sources [51].

The performance allocation of the sub-channels at OFDM mode can be accomplished using the comprehensive mathematical application of the IWFA, with the consideration of minimizing computational procedures so that the channel monitoring/recording processes of the software-controlled radio equipment can still be handled by the built-in processors.

Cutting-age control processors, end-to-end solutions, power supply sources (built-in hardware) and software components capable of multiplying the capabilities of these elements in the public market and military radio equipment enhance the capabilities of these elements by improved mathematical procedures for optimizing the use of electromagnetic spectrum. Based on my current research it is not known whether OFDM technology would have been used to meet broadband military communications needs.

This is why I consider as a basic principle that further (military) technical innovation research and procedures should be investigated to support complex operational applications to meet the needs of broadband data transmission especially in support of rapid reaction units and special operations forces.

## BIBLIOGRAPHY

[1] András Lapsánszky (editor): Hírközlés-szabályozás, hírközlés-igazgatás hazánkban és az Európai Unióban, Wolters Kluver kiadó, 2013. október 31. ISBN 978 96329 52710

[2] Dr. László Pados: A műsorszórás frekvencia lehetőségeinek változásai (előadás), Nemzeti Média és Hírközlési Hatóság, 2013. február 26.

[3] 7/2015. (XI. 13.) NMHH rendelet a nemzeti frekvencia felosztásról, valamint a frekvenciasávok felhasználási szabályairól

[4] Krisztián Németh: Távközlő hálózatok és szolgáltatások (előadás) Budapesti Műszaki és Gazdaságtudományi Egyetem Távközlési és Médiainformatikai Tanszék, 2009. november 11.

[5] Roland Szabó: Hálózatok elemei, tervezése, méretezése, Nemzeti Szakképzési és Felnőttképzési Intézet, number of edition: SzT-009-50

[6] Dr. Nyakóné Dr. Juhász Katalin, Dr. György Terdik, Piroska Biró, Dr. Zoltán Kátai: Bevezetés az informatikába, Debreceni Egyetem Informatikai Kar Kempelen Farkas Hallgatói Információs Központ 2011. május 18.

[7] Zoltán Rajnai: A tábori alaphírhálózat vizsgálata és digitalizálásának lehetőségei egyes NATO tagországok kommunikációs rendszereinek tükrében, doktori (PhD) értekezés, Zrínyi Miklós Nemzetvédelmi Egyetem, Budapest 2001.

[8] András Németh: A mobil szolgáltatók hálózatainak felhasználása, fejlesztési lehetőségei és alternatív megoldások a katasztrófavédelmi kommunikáció területén, doktori (PhD) értekezés, Zrínyi Miklós Nemzetvédelmi Egyetem, Budapest 2008

[9] Krisztián Károly: Lövész zászlóalj kapcsolati rendszereinek vizsgálata hálózatelemzési módszerekkel 1. rész, Hadmérnök, IX. évfolyam 3. szám – 2014. szeptember, Nemzeti Közszolgálati Egyetem Budapest, ISSN 1788 1919

[10] Krisztián Károly: Lövész zászlóalj kapcsolati rendszereinek vizsgálata hálózatelemzési módszerekkel 2. rész, Hadmérnök, IX. évfolyam 4. szám – 2014. szeptember, Nemzeti Közszolgálati Egyetem Budapest, ISSN 1788 1919

[11] Dr. Szilárd Aradi: I+K Technológiák – Digitális adatátviteli alapfogalmak (előadás), Budapesti Műszaki és Gazdaságtudományi Egyetem Közlekedésautomatizálási Tanszék, 2016. november 30.

[12] Dr. Jacob Sharonony: Introduction to Wireless MIMO – Theory and Applications (előadás), Network Technologies Division Center of Excellence in Wireless&IT Stony Book University, 15 November 2006.

[13] Poloha, Wieser, Haring, Majer: Mathematical models of MIMO channel, University of Žilina Department of Telecommunications and Multimedia SK-01026

[14] Balogh, Kollár, Péceli, Németh, Sujbert: Digitális jelfeldolgozás, Tansegédlet MIT-VIMM 4084-01 Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Méréstechnika és Információs Rendszerek Tanszék, 2008. szeptember 17.

[15] Csurgai-Horváth, Héder, Singliar, Sinka, Szládek: Csatornamodellek – Fadinggel terhelt csatornák vizsgálata 1, Mérési Segédlet, Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Szélessávú Hírközlés és Villamosságtan Tanszék Számítógép laboratórium, 2005-2016.

[16] Lajos Nagy: Determinisztikus beltéri hullámterjedési modellek, Híradástechnika folyóirat LXII. Évfolyam 2007/3, ISSN 0018 2028

[17] S. Yee Kane: Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media, IEEE Transactions on Antennas and Propagation, 28 January 1966

[18] David Tse: Optimal power allocation over parallel Gaussian Broadcast Channel, University of California at Berkeley Department of Electrical Engineering and Computer Sciences, International Symposium on Information Theory, Ulm Germany 1997.

[19] Dr. László Pap: Hírközléselmélet I. Tansegédlet, Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Szélessávú Hírközlés és Villamosságtan Tanszék, 2009. április 10.

[20] Utasi, Czúni: Valós idejű mozgásdetektálás módosított Mixture of Gaussian eljárással, Pannon Egyetem Képfeldolgozás és Neuroszámítógépek Tanszék In: http://web.eee.sztaki.hu/~ucu/utasiKEPAF07.pdf Downloading: 2018. december 25.

[21] Lóránt Kovács: Adaptív csatornakiegyenlítő algoritmusok vezeték nélküli hálózatok teljesítőképességének növelésére, Doktori (PhD) értekezés tervezete, Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Híradástechnika Tanszék 2007.

[22] Faragó, Havasi, Horváth: Numerical solution of the Maxwell equations in time-varying media using Magnus expansion, Central European Journal of Mathematics, 9 July 2011, DOI: 10.2478/s11533-011-0074-3 In: https://eudml.org/doc/269193 Downloading: 2018. december 11.

[23] Dr. Villő Csiszár: Bevezetés az információelméletbe, a Gauss-csatorna modellje, Eötvös Loránd Tudományegyetem Természettudományi Kar Matematikai Intézet Valószínűségelméleti és Statisztikai Tanszék, 2011. május 21. In: http://www.math.elte.hu/~mori/infjegyzet.pdf Downloading: 2018. november 30.

[24] L. Pau: Summary introduction to Wireless LTF 4G architecture and key business implications In: http://www.cost605.org/cost605school2011/1-LF-Pau-LTE.pdf Downloading: 2018. november 28.

[25] Alcatel-Lucent: The LTE Network Architecture – Strategic White Paper 2009 In: http://www.cse.unt.edu/~rdantu/FALL_2013_WIRELESS_NETWORKS/LTE_Alcatel_White_Paper.pdf Downloading: 2018. november 27.

[26] Endre Horváth: Amit az LTE technológiáról tudni kell, Internet-cikk, 2011. november 30. In: http://infoter.eu/cikk/amit_az_lte_technologiarol_tudni_kell Downloading: 2018. december 11.

[27] Patrik Paksy: Kommunikációs Hálózatok 2, Előadás, Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Távközlési és Médiainformatikai Tanszék, 2018. február 27. Inhttp://w3.tmit.bme.hu/kh2/kh2-04_mobil_4g_5g_annotated.pdf Downloading: 2018. március 31.

[28] Joseph Mitola III: Cognitive radio: Making software radios more personal, IEEE Personal Communications August 1999, ISSN 1070 9916

[29] M. Sherman: IEEE standards supporting cognitive radio and networks, dynamic spectrum access, and coexsistence, IEEE Communications Magazine July 2008, ISSN 0163 6804

[30] Ákos Sebestyén: A DVB-T rádiófrekvenciás visszirányú megoldásainak bemutatása, Híradástechnika LIX. évfolyam 2004/7, ISSN 0018 2028

[31] Dr. László Pap: Az új mobil technológiák – az LTE és a kognitív rádió alkalmazások műszaki összefüggései, hatásai, Pro-M Zrt. előadás, Budapest 2014. október 26.

[32] Kollár, Varga, Horváth: Modern, többvivős rendszerek kognitív rádiós alkalmazásokban, Híradástechnika LXVI. évfolyam 2011/3. ISSN 0018 2028

[33] Dr. Árpád Dárdai: Ortogonális frekvenciaosztású többszörös hozzáférés, Híradástechnika LIX. évfolyam 2004/4. ISSN 0018 2028]

[34] Kollár-Horváth: PAPR reduction of FBMC by clipping and its iterative compensation, Journal of Computer Networks and Communications Hindawi Publishing Co. Volume 2012 ISSN 2090 7141

[35] Jadhav, Mujawar, Pise: Optimal and Water-Filling Algorithm approach for power allocation in OFDM based Cognitive Radio System, International Journal of Engineering and Technology, Volume 10 Number 1, ISSN 0974 3154

[36] Le Nir, Scheers: Multiple Input Multiple Output Iterative Water-filling Algorithm for multiple broadcast networks distributing only common information, Dept. Communications, Information System&Sensors (CISS) Royal Military Academy Brussels Belgium

[37] Qi, Lang: An efficient Water-filling Algorithm for power allocation in OFDM-based Cognitive Radio Systems, CSE Conference and Workshop Paper, Computer Science and Engineering University of Nebraska-Lincoln, International Conference on System and Informatics (ICSAI 2012) 978-1-4673-0199-2/12, Digital Object Identifier: 10.1109.ICSAI.2012.6223460

[38] Cheng, Verdu: Gaussian multi-access channels with ISI: Capacity region and multiuser water-filling, IEEE Transactions and Information Theory Volume 39 No. 3, 1993 ISSN 0018 9448

[39] David Tse: Capacity region of the multi-access Fading Channel under dynamic resource allocation and polymatroid optimalization, scientific presentation IEEE Information Theory Workshop Haifa 1996

[40] Dr. Tamás Nagy: Konvex függvény, Kvázikonvex függvény, Tansegédlet Miskolci Egyetem Alkalmazott Matematika Tanszék, Miskolc 2012

[41] Babák Bence: Konvexitás - Szélsőérték, Szakdolgozat (Matematika BSc), Eötvös Loránd Tudományegyetem Természettudományi Kar Analízis Tanszék, Budapest 2014.

[42] Bansal, Hossain, Bhargava: Optimal and suboptimal power allocation schemes for OFDM-based Cognitive Radio System, IEEE Transactions and Information Theory Volume 7, November 2008, ISSN 0018 9448

[43] Yan, Ren: Power allocation algorithm for OFDM based Cognitive Radio System, Conference presentation, IEEE International Conference on Wireless Communications Networking and Mobile Computing (WiCOM) 2010

[44] Li: Lagrange Interpolation Formula, Mathematical Excalibur Volume 15 Number 2, July-September 2010, ISSN 2284-5569

[45] Simon Malham: An introduction to Lagrangian and Hamiltonian mechanics, 23 August 2016 In: http://www.macs.hw.ac.uk/~simonm/mechanics.pdf Downloading: 2018. november 12.

[46] Ali Hussein: The Lagrange Interpolation Polynomial for Neutral Network Learning, International Journal of Computer Science and Network Security (IJCSNS) review, Volume 11 No. 3, March 2011, ISSN 1738 7906

[47] Kuhn, Tucker: Nonlinear Programming, Berkeley Symposium on Mathematical Statistic and Problems, University of California Press 1951 In: https://projecteuclid.org/euclid.bsmsp/1200500249 Downloading: 2018. december 02.

[48] H-C Wu: The Karush–Kuhn–Tucker optimality conditions in an optimization problem with interval-valued objective function, European Journal of operational research Volume 176 Issue 1, 01 January 2007, ISSN 0377 2217 In: https://www.sciencedirect.com/science/article/abs/pii/S0377221705007022 Downloading: 2018. december 24.

[49] Dempe, Zemkoho: On the Karush–Kuhn–Tucker reformulation of the bilevel optimization problem, Nonlinear analysis: Theory, Methods and Applications Volume 75 Issue 3, February 2012, ISSN 0362 546X In: https://www.sciencedirect.com/science/article/pii/S0362546X1100407X Downloading: 2018. december 24.

[50] Tapia, Trosset: An Extension of the Karush–Kuhn–Tucker Necessity Conditions to Infinite Programming, Society for Industrial and Applied Mathematics Volume 36 Issue 1, 21 October 1991, 10.1137/1036001 In: https://epubs.siam.org/doi/abs/10.1137/1036001 Downloading: 2018. december 24.

[51] Tünde Balog: Az OFDM hozzáférési technika alkalmazási lehetőségeinek vizsgálata, Diplomaterv, Budapesti Műszaki és Gazdaságtudományi Egyetem Villamos és Informatikai Kar Híradástechnikai Tanszék Mobil Távközlési Laboratórium 2002 In: http://docplayer.hu/12708337-Az-ofdm-hozzaferesi-technika-alkalmazasi-lehetosegeinek-vizsgalata.html Downloading: 2018. december 27.